All work must be shown to be awarded full credit.
Provide exact solutions to all problems, unless otherwise stated. A scientific calculator is allowed.

Student Name: KEY **ID:**________________________
Instructor: **Mundy-Castle**

Exam Score: ________

1) Find the following limits analytically.

a) $\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}$

$$\lim_{x\to 0}\frac{(\sqrt{4+x}-2)}{x}\frac{(\sqrt{4+x}+2)}{(\sqrt{4+x}+2)}$$

$$= \lim_{x\to 0}\frac{4+x-4}{x(\sqrt{4+x}+2)} = \lim_{x\to 0}\frac{x}{x(\sqrt{4+x}+2)} = \lim_{x\to 0}\frac{1}{\sqrt{4+x}+2}$$

$$= \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

b) $\lim_{x\to 4}\frac{x^2-5x+4}{x^2-2x-8} = \lim_{x\to 4}\frac{(x-4)(x-1)}{(x-4)(x+2)}$

$$= \lim_{x\to 4}\frac{x-1}{x+2} = \frac{3}{6} = \frac{1}{2}$$

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2) Find the constants a and b such that $f(x) = \begin{cases} 2a\ , & x \le -1 \\ x-1, & -1 < x < 3 \\ -2b\ , & x \ge 3 \end{cases}$ is continuous on the entire real line.

Need $\lim_{x\to -1} f(x)$ to exist, so we need $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (2a) = 2a$

so $2a = -2$, $a = -1$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (x-1) = -1-1 = -2$

Also need $\lim_{x\to 3} f(x)$ to exist, so we need $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x-1) = 3-1 = 2$

so $-2b = 2$, $b = -1$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-2b) = -2b$

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3) Find the derivative.

a) $y = \frac{3(1-\cos x)}{2\sin x} = \frac{3}{2}\left(\frac{1-\cos x}{\sin x}\right) = \frac{3}{2}\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$

$= \frac{3}{2}\left(\csc x - \cot x\right)$

$\frac{dy}{dx} = \frac{3}{2}\left(-\csc x \cot x + \csc^2 x\right)$

b) $h(x) = \frac{3}{\sqrt{4x^2-x}} = 3\left(4x^2-x\right)^{-1/2}$

$h'(x) = -\frac{3}{2}\left(4x^2-x\right)^{-3/2}\left(8x-1\right) = \frac{-3(8x-1)}{2(4x^2-x)^{3/2}}$

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4) Find dy/dx by implicit differentiation: $2\sin x \cos y = 1$

$$\frac{d}{dx}\left(2\sin x \cos y\right) = \frac{d}{dx}(1)$$

$$2\cos x \cos y + 2\sin x(-\sin y)y' = 0$$

$$-2y' \sin x \sin y = -2\cos x \cos y$$

$$y' = \frac{\cos x \cos y}{\sin x \sin y} = \cot x \cot y$$

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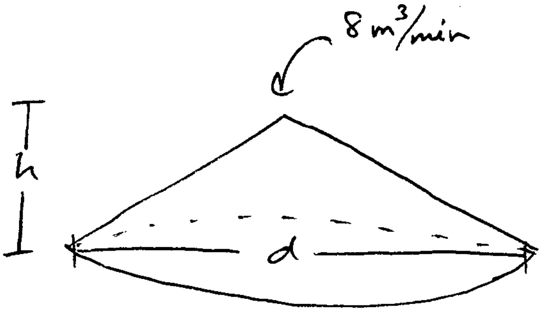
5) The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 8 cubic meters per minute. The diameter of the base of the cone is approximately two times the height. At what rate is the height of the pile changing when the pile is 5 meters high?



Given: $\frac{dV}{dt} = 8 \text{ m}^3/\text{min}.$

Want: $\frac{dh}{dt}$ when $h = 5\text{m}$

$d = 2h$

$2r = 2h$, so $r = h$

Relation: $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (h^2) h = \frac{1}{3}\pi h^3$

Differentiate: $\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

so $\frac{dh}{dt} = \frac{1}{\pi h^2} \cdot \frac{dV}{dt}$

$$\left.\frac{dh}{dt}\right|_{h=5} = \frac{1}{\pi (5)^2}(8) = \frac{8}{25\pi} \simeq 0.102 \text{ m/min.}$$